The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 000000 0000	Conclusions

A naturalistic case in favour of the Generic Multiverse with a core

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Paris Lodron Universität Salzburg

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The Generic Multiverse with a core			Conclusions
	00 000 0000000	00 000000 0000	

Structure of the Presentation

1 Introduction

- 2 The Generic Multiverse with a core
- 3 A first argument for the GM_H
 - Restrictiveness
 - Refining of the definitions
- 4 A further refinement
 - Solving Hamkins' problems
 - A second argument for the GM_H

5 Conclusions



Introduction	The Generic Multiverse with a core			Conclusions
● 0000		00 000 0000000	00 000000 0000	

Structure of the Presentation

1 Introduction

- 2 The Generic Multiverse with a core
- A first argument for the *GM_H* Restrictiveness
 - Refining of the definitions
- 4 A further refinement
 - Solving Hamkins' problems
 - A second argument for the GM_H

5 Conclusions

Introduction	The Generic Multiverse with a core			Conclusions
0000		00 000 0000000	00 000000 0000	

The multiverse conception in set theory

- The naturalistic approach
- A brief sketch of the argument



Introduction	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
0000		00 000 0000000	00 000000 0000	

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Introduction	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
0000		00 000 0000000	00 000000 0000	

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Introduction 00000	The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 000000 0000	Conclusions

Multiverse conceptions in set theory

The broad multiverse

All the possible models of all possible collections of axioms are part of the multiverse.

The generic multiverses

This multiverse is formed by all the models of ZFC(+LCs)obtained by set forcing. Then, we differentiate between universes using a strong logic (an idea owed to Woodin, from now on GM_{Ω}) or supposing the existence of a core (an idea owed to Steel, that is the GM_H).

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The Hyperuniverse

The collection of all countable transitive models of ZFC

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In favour of the GM_H

Introduction 00000	The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 000000 0000	Conclusions

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Introduction 00000	The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 000000 0000	Conclusions

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Introduction 000●0	The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 000000 0000	Conclusions 00

Naturalism in Philosophy of Mathematics

UNIFY

Our framework should be *foundational*: we need an arena in which all mathematical phenomena are represented.

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In our framework there should be as many objects as possible.



Introduction 000●0	The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 000000 0000	Conclusions 00

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Introduction 000●0	The Generic Multiverse with a core	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 000000 0000	Conclusions 00

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Introduction	The Generic Multiverse with a core			Conclusions
00000		00 000 0000000	00 000000 0000	

The main argument

• The GM_H maximizes the number of isomorphisms types available;

• Moreover, classic set theory ZFC is restrictive over the GM_{H} ;

▶ Thus, the *GM_H* strongly maximizes over *ZFC*.



Introduction	The Generic Multiverse with a core			Conclusions
00000		00 000 0000000	00 000000 0000	

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Introduction 0000●	The Generic Multiverse with a core	A first argument for the <i>GM_H</i> 00 000 0000000	A further refinement 00 000000 0000	Conclusions 00

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
● 0000	00 000 0000000	00 000000 0000	

Structure of the Presentation

1 Introduction

2 The Generic Multiverse with a core

- A first argument for the *GM_H* Restrictiveness
 - Refining of the definitions
- 4 A further refinement
 - Solving Hamkins' problems
 - A second argument for the GM_H

5 Conclusions

	The Generic Multiverse with a core	A first argument for the <i>GM_H</i>		Conclusions
00000	0000	00 000 0000000	00 000000 0000	00

The Generic Multiverse with a core (GM_H)

Definition of the core

The core of the multiverse is the collection of all the statements that are true in *every* universe of the multiverse.

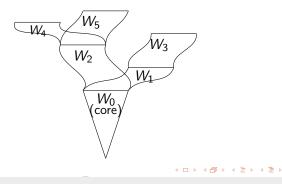


	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	0000	00 000 0000000	00 000000 0000	00

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The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> 00 0000000	A further refinement 00 000000 0000	Conclusions
	0000000	0000	

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- sets (as usual);
 - worlds;

 this language is expressive enough to state versions of the axioms of ZFC and large cardinals hypotheses;



The Generic Multiverse with a core	A first argument for the <i>GM_H</i> 00 0000 0000000	A further refinement 00 000000 0000	Conclusions

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The Generic Multiverse with a core	A first argument for the <i>GM_H</i> 00 0000 0000000	A further refinement 00 000000 0000	Conclusions

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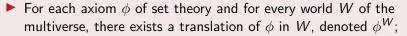
The Generic Multiverse with a core 00●00	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement oo oooooo oooo	Conclusions 00
	0000000	0000	

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	The Generic Multiverse with a core			Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

Axioms



- Every world is a transitive proper class. An object is a set just in case it belongs to some world;
- If W is a world and P ∈ W is a poset, then there is a world of the form W[G] where G is P-generic over W;
- If U is a world, and U = W[G], where G is P-generic over W then W is a world.

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	The Generic Multiverse with a core			Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

Axioms

- For each axiom φ of set theory and for every world W of the multiverse, there exists a translation of φ in W, denoted φ^W;
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	The Generic Multiverse with a core			Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

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	The Generic Multiverse with a core			Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

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The Generic Multiverse with a core 0000●	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 000000 0000	Conclusions
	0000000	0000	

Amalgamation

If U and W are worlds, then there are G and H set generic over them such that W[G] = U[H].

Axiom H

For any sentence ϕ in LST: if ϕ is true, then for some $M = AD^+ + V = L(P(\mathbb{R}))$ such that $\mathbb{R} \cup OR \subseteq M$, $(HOD \cap V_{\Theta})^M \models \phi$. Consequences:

- It implies that the multiverse has a core;
- can be used to study the definability of hierarchies;

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The Generic Multiverse with a core 0000●	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement oo oooooo oooo	Conclusions
		0000	

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The Generic Multiverse with a core 0000●	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement oo oooooo oooo	Conclusions
		0000	

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Z B II R G

The Generic Multiverse with a core 0000●	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 000000 0000	Conclusions
	0000000	0000	

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	• 0 000 0000000	00 000000 0000	

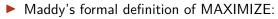
Structure of the Presentation

1 Introduction

- 2 The Generic Multiverse with a core
- A first argument for the GM_H
 Restrictiveness
 - Restrictiveness
 - Refining of the definitions
- 4 A further refinement
 - Solving Hamkins' problems
 - A second argument for the GM_H

5 Conclusions

	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	0 ● 000 0000000	00 000000 0000	00



- Restrictiveness;
- the interpretation relation au;
- fair interpretations;
- Maddy's interpretations;
- A first argument in favour of the GM_H
- Hamkins' counterexamples in Maddy's definitions



	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	0 ● 000 0000000	00 000000 0000	00

Maddy's formal definition of MAXIMIZE:

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	0 ● 000 0000000	00 000000 0000	00

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	0 ● 000 0000000	00 000000 0000	00

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	0 ● 000 0000000	00 000000 0000	00

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	0 ● 000 0000000	00 000000 0000	00

Section outline

Maddy's formal definition of MAXIMIZE:

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	0 ● 000 0000000	00 000000 0000	00

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 •00 0000000	00 000000 0000	

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Restrictiveness

Structure of the Presentation

1 Introduction

- 2 The Generic Multiverse with a core
- A first argument for the *GM_H* Restrictiveness
 - Refining of the definitions
- 4 A further refinement
 - Solving Hamkins' problems
 - A second argument for the GM_H

5 Conclusions

M. de Ceglie In favour of the GM_H

The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 0●0 0000000	00 000000 0000	

Definitions

Let S and T be set theories.

- ▶ $T \leq -$ recaptures S if and only if there is a consistent extension T^* of T such that $S \leq T^*$;
- S weakly ≤ maximizes over T if and only if T ⊲ S and T doesn't ≤ recaptures S, and we write T < get S;</p>
- S strongly ≤ maximizes over T if and only if it weakly ≤ maximizes over T and S ∪ T is inconsistent, and we write T <[≤]_{strong} S;
- T is weakly/strongly ≤ restrictive if and only if there is a set theory T* that is consistent that weakly/strongly ≤ maximizes over T.

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 0●0 0000000	00 000000 0000	

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 0●0 0000000	00 000000 0000	

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 0●0 0000000	00 000000 0000	

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 000 000000	00 000000 0000	

Restrictiveness of ZFC

Proposition

ZFC is strongly restrictive over GM_H .





The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 000 000000	00 000000 0000	

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ZFC is strongly restrictive over GM_H .

- GM_H strongly maximizes over ZFC iff
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 - $ZFC \leq GM_H$
 - there is no set theory T that extends ZFC such that $GM_H riangleq T$
 - **2** $GM_H \cup ZFC$ is inconsistent;

The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 000 000000	00 000000 0000	

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		000	

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 000 000000	00 000000 0000	

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000		00 000 ●000000	00 000000 0000	

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Refining of the definitions

Structure of the Presentation

1 Introduction

- 2 The Generic Multiverse with a core
- A first argument for the GM_H
 Restrictiveness
 - Refining of the definitions
- 4 A further refinement
 - Solving Hamkins' problems
 - A second argument for the GM_H

5 Conclusions

M. de Ceglie In favour of the GM_H

	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	00 000 0●00000	00 000000 0000	00

Translations

A translation $\tau = \langle \delta, \epsilon \rangle$ consists of a \mathcal{L}_{\in} -formula with one variable δ and of a \mathcal{L}_{\in} -formula with two variable:

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$$(x \in y)^{\tau} := \delta(x) \wedge \delta(y) \wedge \epsilon(x, y);$$

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Proposition [Tarski]

A translation $\tau \langle \delta, \epsilon \rangle$ is an *interpretation* of T in S if and only if for every axiom ϕ of T, $S \vdash \phi^{\tau}$.

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	00 000 000000	00 000000 0000	00

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	00 000 000000	00 000000 0000	00

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000	00000	00 000 000000	00 000000 0000	00

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Proposition [Tarski]

A translation $\tau \langle \delta, \epsilon \rangle$ is an *interpretation* of T in S if and only if for every axiom ϕ of T, $S \vdash \phi^{\tau}$.

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
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Translations

A translation $\tau = \langle \delta, \epsilon \rangle$ consists of a \mathcal{L}_{\in} -formula with one variable δ and of a \mathcal{L}_{\in} -formula with two variable:

•
$$(x \in y)^{\tau} := \delta(x) \wedge \delta(y) \wedge \epsilon(x, y);$$

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
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M. de Ceglie In favour of the GM_H

00 00 00 00 00 00 00		The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
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Interpretations

Let S and T be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be a translation of T in S.

► τ is a \in - *interpretation* of T in S if and only $S \vdash \forall x \forall x [(x \in y)^{\tau} \rightarrow x \in y];$

τ is a transitive interpretation of T in S if and only if τ is a ∈
 - interpretation and S⊢ ∀x∀y[(x ∈ y ∧ δ(y)) → δ(x)];

 τ is an inner model interpretation of T in S if and only if τ is
 a transitive interpretation of T in S and S ⊢ ∀α[δ(α)].

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	00000	00000	000	 00

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	00000	00000	000	 00

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 000 000●000	00 000000 0000	

Fair interpretations

Inner model interpretation

Let S and T be set theories. $\tau = \langle \delta, \epsilon \rangle$ is a truncated inner model interpretation of T in S if and only if τ is a transitive interpretation of T in S and $S \vdash Inacc(\kappa) \land \forall \alpha [\alpha < \kappa \leftrightarrow \delta(\alpha)]$.

Fair interpretation

Let S and T be set theories. τ is a *fair interpretation* of T in S if and only if either τ is a inner model interpretation or is a truncated inner model interpretation (in symbols $T \leq_{fair} S$).

The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 000 000●000	00 000000 0000	

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 000 0000●00	00 000000 0000	

Maddy interpretations

Maximizing interpretation

Let S and T be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be an interpretation of T in S. τ is a maximizing interpretation of T in S if and only if $S \vdash Newlso(\tau)$.

Maddy interpretation

Let S and T be set theories. τ is a *Maddy interpretation* of T in S if and only if τ is a fair and maximizing interpretation of T in S. If this is the case, we write $T \leq_{fair} S$.

The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 000 0000●00	00 000000 0000	

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The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
	00 000 0000000	00 000000 0000	

A second proposition

Proposition

 $ZFC \trianglelefteq_{fair} GM_H.$

 GM_H Maddy interprets ZFC iff there exists an interpretation τ of ZFC in GM_H such that ZFC $\leq_{fair} GM_H$;

 $\blacksquare \tau \text{ is a maximizing interpretation of } ZFC \text{ in } GM_H \text{ iff} \\ GM_H \vdash Newlso(\tau);$

 $\boxtimes au$ is a fair interpretation of ZFC in GM_{H}



00000 00000 00 00 00 000 000000	The Generic Multiverse with a core	A first argument for the GM_H	Conclusions
0000000 00000		000	

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00000 00000 00 00 00 000 000000	The Generic Multiverse with a core	A first argument for the GM_H	Conclusions
0000000 00000		000	

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
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Hamkins' counterexamples

► The relation ≤_{Maddy} defined by Maddy has the following structural problems:

the disjunction is not an upper bound;

it is not transitive.

Hamkins' solution, viz. to replace ⊴_{Maddy} with a transitive interpretation relation, is not acceptable.

see Hamkins (2013).

	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
00000		00 000 000000●	00 000000 0000	

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
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	The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
00000	00000	00 000 0000000	•• •••••• •••••	00

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Structure of the Presentation

1 Introduction

- 2 The Generic Multiverse with a core
- A first argument for the *GM_H*Restrictiveness
 - Refining of the definitions
- 4 A further refinement
 - Solving Hamkins' problems
 - A second argument for the GM_H

5 Conclusions

	The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
00000	00000	00 000 0000000	0 ● 000000 0000	00

Section's outline

Definitions to avoid Hamkins' counterexamples;

The new, and final, argument in favour of the GM_H.



	The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
00000	00000	00 000 0000000	0 ● 000000 0000	00

Section's outline

- Definitions to avoid Hamkins' counterexamples;
- The new, and final, argument in favour of the GM_H .

The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
	00 000 0000000	00 ●00000 00000	

Structure of the Presentation

1 Introduction

- 2 The Generic Multiverse with a core
- A first argument for the *GM_H* Restrictiveness
 Refining of the definitions
- 4 A further refinement
 - Solving Hamkins' problems
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5 Conclusions

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The Generic Multiverse with a core	A first argument for the <i>GM_H</i>	A further refinement	Conclusions
	00 000 0000000	00 0●0000 0000	

Possibly truncated interpretation τ (1)

Let $\tau = \langle \delta, \epsilon \rangle$ be a translation and rk be the Mirimanoff rank function. Then we write $\delta_{\alpha}(y)$ for $\delta(y) \wedge rk(y) < \alpha$ and τ_{α} for $\langle \delta_{\alpha}, \epsilon \rangle$.

Possibly truncated interpretation

Let *S* and *T* – where *T* is *BST* + A_0, \ldots, A_n – be set theories. A translation $\tau = \langle \delta, \epsilon \rangle$ is a *possibly truncated interpretation* of *T* in *S* if and only if

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▶ $S \vdash \phi^{\tau}$ for every axiom of T;

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	The Generic Multiverse with a core	A first argument for the <i>GM_H</i>	A further refinement	Conclusions
		00 000 0000000	00 0●0000 0000	

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The Generic Multiverse with a core	A first argument for the <i>GM_H</i>	A further refinement	Conclusions
	00 000 0000000	00 0●0000 0000	

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	The Generic Multiverse with a core	A first argument for the <i>GM_H</i>	A further refinement	Conclusions
		00 000 0000000	00 0●0000 0000	

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	The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

Possibly truncated interpretations τ (2)

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The Generic Multiverse with a core	A first argument for the <i>GM_H</i>	A further refinement	Conclusions
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The Generic Multiverse with a core	A first argument for the <i>GM_H</i>	A further refinement	Conclusions
	00 000 0000000	00 000000 0000	

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The Generic Multiverse with a core	A first argument for the <i>GM_H</i>	A further refinement	Conclusions
	00 000 0000000	00 000000 0000	

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	The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

Maximizing possibly truncated interpretation

Let S and T - T being $BST + A_0, \ldots, A_n$ – be set theories. A translation $\tau = \langle \delta, \epsilon \rangle$ is a maximizing possibly truncated interpretation of T in S if and only if

 $\blacktriangleright S \vdash (A^{\tau} \land Newlso(\tau) \lor (\exists \kappa [nacc(\kappa)^{\tau} \land (A^{\tau_{\kappa}} \land Newlso(\tau_{\kappa}))])).$



The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
	00 000 0000000	00 000000 0000	

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	The Generic Multiverse with a core		A further refinement	Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

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	The Generic Multiverse with a core	A first argument for the <i>GM_H</i>	A further refinement	Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

Maddy^{*} interpretation

Let S and T - T being $BST + A_0, \ldots, A_n$ – be set theories and let τ be a possibly truncated interpretation of T in S. τ is a $Maddy^*$ interpretation if and only if

is a possibly truncated inner model interpretation;

▶ is a maximizing possibly truncated interpretation. If this is the case, we say that *S* Maddy^{*} interpretates *T* and we write $T \leq_{Maddy^*} S$.

	The Generic Multiverse with a core	A first argument for the <i>GM_H</i>	A further refinement	Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

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If this is the case, we say that $S \ Maddy^*$ interpretates T and we write $T \trianglelefteq_{Maddy^*} S$.



	The Generic Multiverse with a core	A first argument for the <i>GM_H</i>	A further refinement	Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

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	The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
00000	00000	00 000 0000000	00 000000 0000	00

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The Generic Multiverse with a core		A further refinement	Conclusions
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Solving Hamkins' problems

Solving Hamkins' problems

Proposition [Incurvati, Löwe]

Both \trianglelefteq_{ptim} and $\trianglelefteq_{Maddy^*}$ are monotone.

Proposition [Incurvati, Löwe

If $T \leq_{ptim} BST + A$ and $T \leq_{ptim} BST + B$, then $T \leq_{ptim} BST + A \lor B$. The same holds for \leq_{Maddy^*} .

Proposition [Incurvati, Löwe

Both \leq_{ptim} and \leq_{Maddy^*} are transitive.

The Generic Multiverse with a core		A further refinement	Conclusions
	00 000 0000000	00 00000● 00000	

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The Generic Multiverse with a core		A further refinement	Conclusions
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The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
	00 000 0000000	00 000000 ●000	

Structure of the Presentation

1 Introduction

- 2 The Generic Multiverse with a core
- A first argument for the *GM_H* Restrictiveness
 Refining of the definitions
- 4 A further refinement
 - Solving Hamkins' problems
 - A second argument for the GM_H

The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
	00 000 0000000	00 000000 0●00	

A new proposition

Proposition

 $ZFC \trianglelefteq_{Maddy^*} GM_H.$

 GM_H Maddy* interpretates ZFC if and only if there exists a translation τ of ZFC in GM_H that is a



Introduction	The Generic Multiverse with a core			
00000	00000	00 000 0000000	00 000000 00000	00

A new proposition

Proposition

 $ZFC \trianglelefteq_{Maddy^*} GM_H.$

- GM_H $Maddy^*$ interpretates ZFC if and only if there exists a translation τ of ZFC in GM_H that is a
 - **1** a possible truncated inner model interpretation of *ZFC* in GM_H ;
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	The Generic Multiverse with a core	A first argument for the GM_H	A further refinement	Conclusions
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The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 000000 0000	Conclusions

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The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> 00 000 0000000	A further refinement	Conclusions

Sketch of the proof (1)

1 ZFC $\trianglelefteq_{Maddv^*}$ GM_H \iff ZFC \trianglelefteq_{ptim} GM_H

• τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;

• τ is a possible truncated \in - interpretation of ZFC in GM_H

 $(\alpha)^{\delta}_{\mu} = M_{0} = M_{0} = M_{0}$ $(\alpha)^{\delta}_{\mu} = M_{0} = M_{0} = M_{0}$



The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 00000 0000	Conclusions

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 $= GM_H \vdash \forall x \forall y [(x \in y \land \delta(y)) \rightarrow \delta(x)].$

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 \blacksquare ZFC \leq_{Modely} $GM_H \iff \exists \tau$ such that

The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement ○○ ○○○○○○ ○○●○	Conclusions 00

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The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 00000 0000	Conclusions 00

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The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement ○○ ○○○○○○ ○○●○	Conclusions 00

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The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 00000 0000	Conclusions 00

M. de Ceglie In favour of the *GM_H*

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 $= GM_{12} \vdash (A^{*} \wedge Newlso(r_{1}) \vee (\exists \kappa [hacc(\kappa)^{*} \wedge (A^{*} \wedge Newlso(r_{n}))]))$

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The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 00000 0000	Conclusions 00

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 $GM_H \vdash \phi^{\tau}$ for every axiom ϕ of ZFC

 $GM_H \vdash (A^{e} \land Newlso(\tau) \lor (\exists \kappa [nacc(\kappa)^{e} \land (A^{e} \land Newlso(\tau_{\kappa})))$

The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 00000 0000	Conclusions 00

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The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement 00 00000 0000	Conclusions 00

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The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> oo ooo ooooooo	A further refinement ○○ ○○○○○○ ○○●○	Conclusions 00

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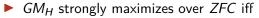
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IntroductionThe Generic Multiverse with a core0000000000	A first argument for the <i>GM_H</i> 00 000	A further refinement	Conclusions 00
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The final proposition

Proposition

ZFC is strongly restrictive over GM_H .



GM_H weakly maximizes over *ZFC* if

- ZFC ⊴_{Maddy*} GM_H;
- there is no set theory T that extends ZEC such that
- GM_H ⊴_{Maday}≁ T

2 $GM_H \cup ZFC$ is inconsistent;



The Generic Multiverse with a core	A first argument for the <i>GM_H</i> oo ooo	A further refinement	Conclusions
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The Generic Multiverse with a core	A first argument for the <i>GM_H</i> oo ooo	A further refinement	Conclusions
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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
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Structure of the Presentation

1 Introduction

- 2 The Generic Multiverse with a core
- A first argument for the *GM_H* Restrictiveness
 - Refining of the definitions
- 4 A further refinement
 - Solving Hamkins' problems
 - A second argument for the GM_H

5 Conclusions

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	The Generic Multiverse with a core	A first argument for the GM_H		Conclusions
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- Assuming MAXIMIZE, the GM_H fares better than the classic ZFC, since it provides more isomorphisms types.
- This is true of every type of multiverse.
- ▶ But the *GM_H* can interpret *ZFC*;
- In conclusion, the GM_H seems naturalistic better than ZFC.



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The Generic Multiverse with a core 00000	A first argument for the <i>GM_H</i> 00 000 0000000	A further refinement 00 000000 0000	Conclusions ○●

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