A naturalistic case in favour of the Generic Multiverse with a core

Matteo de Ceglie
decegliematteo@gmail.com

Paris Lodron Universität Salzburg

1^{st} Forcing Networking Project Conference
20^{th} September 2018

Structure of the Presentation

Structure of the Presentation

Section outline

- The multiverse conception in set theory
- The naturalistic approach
- A brief sketch of the argument

Section outline

- The multiverse conception in set theory
- The naturalistic approach
- A brief sketch of the argument

Section outline

- The multiverse conception in set theory
- The naturalistic approach
- A brief sketch of the argument

Multiverse conceptions in set theory

The *broad* multiverse

All the possible models of all possible collections of axioms are part of the multiverse.

The generic multiverses

This multiverse is formed by all the models of $ZFC(+LCs)$ obtained by set forcing. Then, we differentiate between universes using a strong logic (an idea owed to Woodin, from now on GM_Ω) or supposing the existence of a core (an idea owed to Steel, that is the GM_H).

The Hyperuniverse

The collection of all countable transitive models of ZFC.

Multiverse conceptions in set theory

The *broad* multiverse

All the possible models of all possible collections of axioms are part of the multiverse.

The generic multiverses

This multiverse is formed by all the models of $ZFC(+LCs)$ obtained by set forcing. Then, we differentiate between universes using a strong logic (an idea owed to Woodin, from now on GM_Ω) or supposing the existence of a core (an idea owed to Steel, that is the GM_H).

The Hyperuniverse

The collection of all countable transitive models of ZFC.

Multiverse conceptions in set theory

The *broad* multiverse

All the possible models of all possible collections of axioms are part of the multiverse.

The generic multiverses

This multiverse is formed by all the models of $ZFC(+LCs)$ obtained by set forcing. Then, we differentiate between universes using a strong logic (an idea owed to Woodin, from now on GM_Ω) or supposing the existence of a core (an idea owed to Steel, that is the GM_H).

The Hyperuniverse

The collection of all countable transitive models of ZFC.

Naturalism in Philosophy of Mathematics

UNIFY

Our framework should be *foundational*: we need an arena in which all mathematical phenomena are represented.

MAXIMIZE

In our framework there should be as many objects as possible.

Naturalism in Philosophy of Mathematics

UNIFY

Our framework should be *foundational*: we need an arena in which all mathematical phenomena are represented.

MAXIMIZE

In our framework there should be as many objects as possible.

Naturalism in Philosophy of Mathematics

UNIFY

Our framework should be *foundational*: we need an arena in which all mathematical phenomena are represented.

MAXIMIZE

In our framework there should be as many objects as possible.

The main argument

- The GM_H maximizes the number of isomorphisms types available;
- Moreover, classic set theory ZFC is *restrictive* over the GM_H;
- Thus, the GM_H *strongly maximizes* over ZFC.

The main argument

- The GM_H maximizes the number of isomorphisms types available;
- Moreover, classic set theory *ZFC* is *restrictive* over the GM_H;
- Thus, the GM_H *strongly maximizes* over *ZFC*.

The main argument

- The GM_H maximizes the number of isomorphisms types available;
- Moreover, classic set theory *ZFC* is *restrictive* over the GM_H;
- Thus, the GM_H *strongly maximizes* over *ZFC*.

Structure of the Presentation

The Generic Multiverse with a core (GM_H)

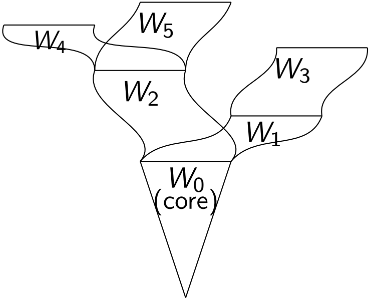
Definition of the *core*

The core of the multiverse is the collection of all the statements that are true in *every* universe of the multiverse.

The Generic Multiverse with a core (GM_H)

Definition of the *core*

The core of the multiverse is the collection of all the statements that are true in *every* universe of the multiverse.



The Multiverse Language *MV* for GM_H

- The usual syntax of the language of set theory, but with two sorts:
  - sets (as usual);
  - *worlds*;
- this language is expressive enough to state versions of the axioms of *ZFC* *and* large cardinals hypotheses;

The Multiverse Language *MV* for GM_H

- The usual syntax of the language of set theory, but with two sorts:
  - sets (as usual);
  - *worlds*;
- this language is expressive enough to state versions of the axioms of *ZFC and* large cardinals hypotheses;

The Multiverse Language *MV* for GM_H

- The usual syntax of the language of set theory, but with two sorts:
  - sets (as usual);
  - *worlds*;
- this language is expressive enough to state versions of the axioms of *ZFC and* large cardinals hypotheses;

The Multiverse Language *MV* for GM_H

- The usual syntax of the language of set theory, but with two sorts:
  - sets (as usual);
  - *worlds*;
- this language is expressive enough to state versions of the axioms of *ZFC and* large cardinals hypotheses;

The Axioms of the GM_H

Axioms

- For each axiom ϕ of set theory and for every world W of the multiverse, there exists a translation of ϕ in W, denoted ϕ^W;
- Every world is a transitive proper class. An object is a set just in case it belongs to some world;
- If W is a world and $\mathbb{P} \in W$ is a poset, then there is a world of the form $W[G]$ where G is $\mathbb{P}$-generic over W;
- If U is a world, and $U = W[G]$, where G is $\mathbb{P}$-generic over W, then W is a world.

UNIVERSITÄT SALZBURG

The Axioms of the GM_H

Axioms

- For each axiom ϕ of set theory and for every world W of the multiverse, there exists a translation of ϕ in W, denoted ϕ^W;
- Every world is a transitive proper class. An object is a set just in case it belongs to some world;
- If W is a world and $\mathbb{P} \in W$ is a poset, then there is a world of the form $W[G]$ where G is $\mathbb{P}$-generic over W;
- If U is a world, and $U = W[G]$, where G is $\mathbb{P}$-generic over W, then W is a world.

UNIVERSITÄT SALZBURG

The Axioms of the GM_H

Axioms

- For each axiom ϕ of set theory and for every world W of the multiverse, there exists a translation of ϕ in W, denoted ϕ^W;
- Every world is a transitive proper class. An object is a set just in case it belongs to some world;
- If W is a world and $\mathbb{P} \in W$ is a poset, then there is a world of the form $W[G]$ where G is $\mathbb{P}$-generic over W;
- If U is a world, and $U = W[G]$, where G is $\mathbb{P}$-generic over W, then W is a world.

The Axioms of the GM_H

Axioms

- For each axiom ϕ of set theory and for every world W of the multiverse, there exists a translation of ϕ in W, denoted ϕ^W;
- Every world is a transitive proper class. An object is a set just in case it belongs to some world;
- If W is a world and $\mathbb{P} \in W$ is a poset, then there is a world of the form $W[G]$ where G is $\mathbb{P}$-generic over W;
- If U is a world, and $U = W[G]$, where G is $\mathbb{P}$-generic over W, then W is a world.

The Axioms of the GM_H (cont.)

Amalgamation

If U and W are worlds, then there are G and H set generic over them such that $W[G] = U[H]$.

Axiom H

For any sentence ϕ in LST: if ϕ is true, then for some $M = AD^+ + V = L(P(\mathbb{R}))$ such that $\mathbb{R} \cup OR \subseteq M$, $(HOD \cap V_\Theta)^M \models \phi$. Consequences:

- It implies that the multiverse has a core;
- can be used to study the definability of hierarchies;
- it is consistent with large cardinals hypotheses.

The Axioms of the GM_H (cont.)

Amalgamation

If U and W are worlds, then there are G and H set generic over them such that $W[G] = U[H]$.

Axiom H

For any sentence ϕ in LST: if ϕ is true, then for some $M = AD^+ + V = L(P(\mathbb{R}))$ such that $\mathbb{R} \cup OR \subseteq M$, $(HOD \cap V_\Theta)^M \models \phi$. Consequences:

- It implies that the multiverse has a core;
- can be used to study the definability of hierarchies;
- it is consistent with large cardinals hypotheses.

The Axioms of the GM_H (cont.)

Amalgamation

If U and W are worlds, then there are G and H set generic over them such that $W[G] = U[H]$.

Axiom H

For any sentence ϕ in LST: if ϕ is true, then for some $M = AD^+ + V = L(P(\mathbb{R}))$ such that $\mathbb{R} \cup OR \subseteq M$, $(HOD \cap V_\Theta)^M \models \phi$. Consequences:

- It implies that the multiverse has a core;
- can be used to study the definability of hierarchies;
- it is consistent with large cardinals hypotheses.

The Axioms of the GM_H (cont.)

Amalgamation

If U and W are worlds, then there are G and H set generic over them such that $W[G] = U[H]$.

Axiom H

For any sentence ϕ in LST: if ϕ is true, then for some $M = AD^+ + V = L(P(\mathbb{R}))$ such that $\mathbb{R} \cup OR \subseteq M$, $(HOD \cap V_\Theta)^M \models \phi$. Consequences:

- It implies that the multiverse has a core;
- can be used to study the definability of hierarchies;
- it is consistent with large cardinals hypotheses.

Structure of the Presentation

UNIVERSITÄT SALZBURG

Section outline

- Maddy's formal definition of MAXIMIZE:
  - Restrictiveness;
  - the interpretation relation τ;
  - fair interpretations;
  - Maddy's interpretations;
- A first argument in favour of the GM_H;
- Hamkins' counterexamples in Maddy's definitions.

Section outline

- Maddy's formal definition of MAXIMIZE:
  - Restrictiveness;
  - the interpretation relation τ;
  - fair interpretations;
  - Maddy's interpretations;
- A first argument in favour of the GM_H;
- Hamkins' counterexamples in Maddy's definitions.

Section outline

- Maddy's formal definition of MAXIMIZE:
  - Restrictiveness;
  - the interpretation relation τ;
  - fair interpretations;
  - Maddy's interpretations;
- A first argument in favour of the GM_H;
- Hamkins' counterexamples in Maddy's definitions.

Section outline

- Maddy's formal definition of MAXIMIZE:
  - Restrictiveness;
  - the interpretation relation τ;
  - fair interpretations;
  - Maddy's interpretations;
- A first argument in favour of the GM_H;
- Hamkins' counterexamples in Maddy's definitions.

Section outline

- Maddy's formal definition of MAXIMIZE:
  - Restrictiveness;
  - the interpretation relation τ;
  - fair interpretations;
  - Maddy's interpretations;
- A first argument in favour of the GM_H;
- Hamkins' counterexamples in Maddy's definitions.

Section outline

- Maddy's formal definition of MAXIMIZE:
  - Restrictiveness;
  - the interpretation relation τ;
  - fair interpretations;
  - Maddy's interpretations;
- A first argument in favour of the GM_H;
- Hamkins' counterexamples in Maddy's definitions.

Section outline

- Maddy's formal definition of MAXIMIZE:
  - Restrictiveness;
  - the interpretation relation τ;
  - fair interpretations;
  - Maddy's interpretations;
- A first argument in favour of the GM_H;
- Hamkins' counterexamples in Maddy's definitions.

Structure of the Presentation

Definitions

Let S and T be set theories.

- $T \trianglelefteq$ - *recaptures* S if and only if there is a consistent extension T^* of T such that $S \trianglelefteq T^*$;
- S *weakly* $\trianglelefteq$ - *maximizes* over T if and only if $T \triangleleft S$ and T doesn't $\trianglelefteq$ - recaptures S, and we write $T <^{\trianglelefteq}_{weak} S$;
- S *strongly* $\trianglelefteq$ - *maximizes* over T if and only if it weakly $\trianglelefteq$ - maximizes over T and $S \cup T$ is inconsistent, and we write $T <^{\trianglelefteq}_{strong} S$;
- T is weakly/strongly $\trianglelefteq$ - *restrictive* if and only if there is a set theory T^* that is consistent that weakly/strongly $\trianglelefteq$ - maximizes over T.

Definitions

Let S and T be set theories.

- $T \trianglelefteq$ - *recaptures* S if and only if there is a consistent extension T^* of T such that $S \trianglelefteq T^*$;
- S *weakly* $\trianglelefteq$ - *maximizes* over T if and only if $T \triangleleft S$ and T doesn't $\trianglelefteq$ - recaptures S, and we write $T <^{\trianglelefteq}_{weak} S$;
- S *strongly* $\trianglelefteq$ - *maximizes* over T if and only if it weakly $\trianglelefteq$ - maximizes over T and $S \cup T$ is inconsistent, and we write $T <^{\trianglelefteq}_{strong} S$;
- T is weakly/strongly $\trianglelefteq$ - *restrictive* if and only if there is a set theory T^* that is consistent that weakly/strongly $\trianglelefteq$ - maximizes over T.

Definitions

Let S and T be set theories.

- $T \trianglelefteq$ - *recaptures* S if and only if there is a consistent extension T^* of T such that $S \trianglelefteq T^*$;
- S *weakly* $\trianglelefteq$ - *maximizes* over T if and only if $T \triangleleft S$ and T doesn't $\trianglelefteq$ - recaptures S, and we write $T <^{\trianglelefteq}_{weak} S$;
- S *strongly* $\trianglelefteq$ - *maximizes* over T if and only if it weakly $\trianglelefteq$ - maximizes over T and $S \cup T$ is inconsistent, and we write $T <^{\trianglelefteq}_{strong} S$;
- T is weakly/strongly $\trianglelefteq$ - *restrictive* if and only if there is a set theory T^* that is consistent that weakly/strongly $\trianglelefteq$ - maximizes over T.

Definitions

Let S and T be set theories.

- $T \trianglelefteq$ - *recaptures* S if and only if there is a consistent extension T^* of T such that $S \trianglelefteq T^*$;
- S *weakly* $\trianglelefteq$ - *maximizes* over T if and only if $T \triangleleft S$ and T doesn't $\trianglelefteq$ - recaptures S, and we write $T <^{\trianglelefteq}_{weak} S$;
- S *strongly* $\trianglelefteq$ - *maximizes* over T if and only if it weakly $\trianglelefteq$ - maximizes over T and $S \cup T$ is inconsistent, and we write $T <^{\trianglelefteq}_{strong} S$;
- T is weakly/strongly $\trianglelefteq$ - *restrictive* if and only if there is a set theory T^* that is consistent that weakly/strongly $\trianglelefteq$ - maximizes over T.

Restrictiveness of *ZFC*

Proposition

ZFC is strongly restrictive over GM_H.

- GM_H strongly maximizes over *ZFC* iff
  1. GM_H weakly maximizes over *ZFC* iff
     - $ZFC \trianglelefteq GM_H$;
     - there is no set theory T that extends *ZFC* such that $GM_H \triangleleft T$
  2. $GM_H \cup ZFC$ is inconsistent;

Restrictiveness of *ZFC*

Proposition

ZFC is strongly restrictive over GM_H.

- GM_H strongly maximizes over *ZFC* iff
  1. GM_H weakly maximizes over *ZFC* iff
     - $ZFC \trianglelefteq GM_H$;
     - there is no set theory T that extends *ZFC* such that $GM_H \trianglelefteq T$
  2. $GM_H \cup ZFC$ is inconsistent;

Restrictiveness of *ZFC*

Proposition

ZFC is strongly restrictive over GM_H.

- GM_H strongly maximizes over ZFC iff
  1. GM_H weakly maximizes over ZFC iff
     - $ZFC \trianglelefteq GM_H$;
     - there is no set theory T that extends ZFC such that $GM_H \trianglelefteq T$
  2. $GM_H \cup ZFC$ is inconsistent;

Restrictiveness of *ZFC*

Proposition

ZFC is strongly restrictive over GM_H.

- GM_H strongly maximizes over *ZFC* iff
  1. GM_H weakly maximizes over *ZFC* iff
     - $ZFC \trianglelefteq GM_H$;
     - there is no set theory T that extends *ZFC* such that $GM_H \trianglelefteq T$
  2. $GM_H \cup ZFC$ is inconsistent;

Restrictiveness of *ZFC*

Proposition

ZFC is strongly restrictive over GM_H.

- GM_H strongly maximizes over *ZFC* iff
  1. GM_H weakly maximizes over *ZFC* iff
     - $ZFC \trianglelefteq GM_H$;
     - there is no set theory T that extends *ZFC* such that $GM_H \trianglelefteq T$
  2. $GM_H \cup ZFC$ is inconsistent;

Structure of the Presentation

Translations

A translation $\tau = \langle \delta, \epsilon \rangle$ consists of a $\mathcal{L}_\in$-formula with one variable δ and of a $\mathcal{L}_\in$-formula with two variable:

- $(x \in y)^\tau := \delta(x) \wedge \delta(y) \wedge \epsilon(x, y)$;
- $(\phi \wedge \psi)^\tau := \phi^\tau \wedge \psi^\tau$;
- $(\neg\phi)^\tau := \neg\phi^\tau$;
- $(\exists x \phi)^\tau := \exists x[\delta(x) \wedge \phi^\tau]$.

Proposition [Tarski]

A translation $\tau\langle \delta, \epsilon \rangle$ is an *interpretation* of T in S if and only if for every axiom ϕ of T, $S \vdash \phi^\tau$.

Translations

A translation $\tau = \langle \delta, \epsilon \rangle$ consists of a $\mathcal{L}_{\in}$-formula with one variable δ and of a $\mathcal{L}_{\in}$-formula with two variable:

- $(x \in y)^{\tau} := \delta(x) \wedge \delta(y) \wedge \epsilon(x, y)$;
- $(\phi \wedge \psi)^{\tau} := \phi^{\tau} \wedge \psi^{\tau}$;
- $(\neg\phi)^{\tau} := \neg\phi^{\tau}$;
- $(\exists x \phi)^{\tau} := \exists x[\delta(x) \wedge \phi^{\tau}]$.

Proposition [Tarski]

A translation $\tau\langle \delta, \epsilon \rangle$ is an *interpretation* of T in S if and only if for every axiom ϕ of T, $S \vdash \phi^{\tau}$.

Translations

A translation $\tau = \langle \delta, \epsilon \rangle$ consists of a $\mathcal{L}_\in$-formula with one variable δ and of a $\mathcal{L}_\in$-formula with two variable:

- $(x \in y)^\tau := \delta(x) \wedge \delta(y) \wedge \epsilon(x, y)$;
- $(\phi \wedge \psi)^\tau := \phi^\tau \wedge \psi^\tau$;
- $(\neg\phi)^\tau := \neg\phi^\tau$;
- $(\exists x \phi)^\tau := \exists x[\delta(x) \wedge \phi^\tau]$.

Proposition [Tarski]

A translation $\tau\langle \delta, \epsilon \rangle$ is an *interpretation* of T in S if and only if for every axiom ϕ of T, $S \vdash \phi^\tau$.

Translations

A translation $\tau = \langle \delta, \epsilon \rangle$ consists of a $\mathcal{L}_\in$-formula with one variable δ and of a $\mathcal{L}_\in$-formula with two variable:

- $(x \in y)^\tau := \delta(x) \wedge \delta(y) \wedge \epsilon(x, y)$;
- $(\phi \wedge \psi)^\tau := \phi^\tau \wedge \psi^\tau$;
- $(\neg\phi)^\tau := \neg\phi^\tau$;
- $(\exists x \phi)^\tau := \exists x[\delta(x) \wedge \phi^\tau]$.

Proposition [Tarski]

A translation $\tau\langle \delta, \epsilon \rangle$ is an *interpretation* of T in S if and only if for every axiom ϕ of T, $S \vdash \phi^\tau$.

Translations

A translation $\tau = \langle \delta, \epsilon \rangle$ consists of a $\mathcal{L}_\in$-formula with one variable δ and of a $\mathcal{L}_\in$-formula with two variable:

- $(x \in y)^\tau := \delta(x) \wedge \delta(y) \wedge \epsilon(x, y)$;
- $(\phi \wedge \psi)^\tau := \phi^\tau \wedge \psi^\tau$;
- $(\neg\phi)^\tau := \neg\phi^\tau$;
- $(\exists x \phi)^\tau := \exists x[\delta(x) \wedge \phi^\tau]$.

Proposition [Tarski]

A translation $\tau\langle \delta, \epsilon \rangle$ is an *interpretation* of T in S if and only if for every axiom ϕ of T, $S \vdash \phi^\tau$.

Translations

A translation $\tau = \langle \delta, \epsilon \rangle$ consists of a $\mathcal{L}_{\in}$-formula with one variable δ and of a $\mathcal{L}_{\in}$-formula with two variable:

- $(x \in y)^\tau := \delta(x) \wedge \delta(y) \wedge \epsilon(x, y)$;
- $(\phi \wedge \psi)^\tau := \phi^\tau \wedge \psi^\tau$;
- $(\neg \phi)^\tau := \neg \phi^\tau$;
- $(\exists x \phi)^\tau := \exists x[\delta(x) \wedge \phi^\tau]$.

Proposition [Tarski]

A translation $\tau\langle \delta, \epsilon \rangle$ is an *interpretation* of T in S if and only if for every axiom ϕ of T, $S \vdash \phi^\tau$.

Interpretations

Let S and T be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be a translation of T in S.

- τ is a $\in$ - *interpretation* of T in S if and only $S \vdash \forall x \forall x[(x \in y)^{\tau} \rightarrow x \in y]$;
- τ is a *transitive interpretation* of T in S if and only if τ is a $\in$ - interpretation and $S \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$;
- τ is an *inner model interpretation* of T in S if and only if τ is a transitive interpretation of T in S and $S \vdash \forall \alpha[\delta(\alpha)]$.

Interpretations

Let S and T be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be a translation of T in S.

- τ is a $\in$ - *interpretation* of T in S if and only $S \vdash \forall x \forall x[(x \in y)^\tau \rightarrow x \in y]$;
- τ is a *transitive interpretation* of T in S if and only if τ is a $\in$ - interpretation and $S \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$;
- τ is an *inner model interpretation* of T in S if and only if τ is a transitive interpretation of T in S and $S \vdash \forall \alpha[\delta(\alpha)]$.

Interpretations

Let S and T be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be a translation of T in S.

- τ is a $\in$ - *interpretation* of T in S if and only $S \vdash \forall x \forall x[(x \in y)^{\tau} \rightarrow x \in y]$;
- τ is a *transitive interpretation* of T in S if and only if τ is a $\in$ - interpretation and $S \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$;
- τ is an *inner model interpretation* of T in S if and only if τ is a transitive interpretation of T in S and $S \vdash \forall \alpha[\delta(\alpha)]$.

Fair interpretations

Inner model interpretation

Let S and T be set theories. $\tau = \langle \delta, \epsilon \rangle$ is a *truncated inner model interpretation* of T in S if and only if τ is a transitive interpretation of T in S and $S \vdash Inacc(\kappa) \wedge \forall \alpha[\alpha < \kappa \leftrightarrow \delta(\alpha)]$.

Fair interpretation

Let S and T be set theories. τ is a *fair interpretation* of T in S if and only if either τ is a inner model interpretation or is a truncated inner model interpretation (in symbols $T \trianglelefteq_{fair} S$).

Fair interpretations

Inner model interpretation

Let S and T be set theories. $\tau = \langle \delta, \epsilon \rangle$ is a *truncated inner model interpretation* of T in S if and only if τ is a transitive interpretation of T in S and $S \vdash Inacc(\kappa) \wedge \forall \alpha[\alpha < \kappa \leftrightarrow \delta(\alpha)]$.

Fair interpretation

Let S and T be set theories. τ is a *fair interpretation* of T in S if and only if either τ is a inner model interpretation or is a truncated inner model interpretation (in symbols $T \trianglelefteq_{fair} S$).

Maddy interpretations

Maximizing interpretation

Let S and T be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be an interpretation of T in S. τ is a maximizing interpretation of T in S if and only if $S \vdash NewIso(\tau)$.

Maddy interpretation

Let S and T be set theories. τ is a *Maddy interpretation* of T in S if and only if τ is a fair and maximizing interpretation of T in S. If this is the case, we write $T \trianglelefteq_{fair} S$.

Maddy interpretations

Maximizing interpretation

Let S and T be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be an interpretation of T in S. τ is a maximizing interpretation of T in S if and only if $S \vdash NewIso(\tau)$.

Maddy interpretation

Let S and T be set theories. τ is a *Maddy interpretation* of T in S if and only if τ is a fair and maximizing interpretation of T in S. If this is the case, we write $T \trianglelefteq_{fair} S$.

A second proposition

Proposition

$ZFC \trianglelefteq_{fair} GM_H$.

GM_H Maddy interprets ZFC iff there exists an interpretation τ of ZFC in GM_H such that $ZFC \trianglelefteq_{fair} GM_H$;

1. τ is a maximizing interpretation of ZFC in GM_H iff $GM_H \vdash NewIso(\tau)$;
2. τ is a fair interpretation of ZFC in GM_H

A second proposition

Proposition

$ZFC \trianglelefteq_{fair} GM_H$.

GM_H Maddy interprets ZFC iff there exists an interpretation τ of ZFC in GM_H such that $ZFC \trianglelefteq_{fair} GM_H$;

1. τ is a maximizing interpretation of ZFC in GM_H iff $GM_H \vdash NewIso(\tau)$;
2. τ is a fair interpretation of ZFC in GM_H

A second proposition

Proposition

$ZFC \trianglelefteq_{fair} GM_H$.

GM_H Maddy interprets ZFC iff there exists an interpretation τ of ZFC in GM_H such that $ZFC \trianglelefteq_{fair} GM_H$;

1. τ is a maximizing interpretation of ZFC in GM_H iff $GM_H \vdash NewIso(\tau)$;
2. τ is a fair interpretation of ZFC in GM_H

Hamkins' counterexamples

- The relation $\trianglelefteq_{Maddy}$ defined by Maddy has the following structural problems:
  - the disjunction is not an upper bound;
  - it is not transitive.
- Hamkins' solution, viz. to replace $\trianglelefteq_{Maddy}$ with a transitive interpretation relation, is not acceptable.
- see Hamkins (2013).

Hamkins' counterexamples

- The relation $\trianglelefteq_{Maddy}$ defined by Maddy has the following structural problems:
  - the disjunction is not an upper bound;
  - it is not transitive.
- Hamkins' solution, viz. to replace $\trianglelefteq_{Maddy}$ with a transitive interpretation relation, is not acceptable.
- see Hamkins (2013).

Hamkins' counterexamples

- The relation $\trianglelefteq_{Maddy}$ defined by Maddy has the following structural problems:
  - the disjunction is not an upper bound;
  - it is not transitive.
- Hamkins' solution, viz. to replace $\trianglelefteq_{Maddy}$ with a transitive interpretation relation, is not acceptable.
- see Hamkins (2013).

Hamkins' counterexamples

- The relation $\trianglelefteq_{Maddy}$ defined by Maddy has the following structural problems:
  - the disjunction is not an upper bound;
  - it is not transitive.
- Hamkins' solution, viz. to replace $\trianglelefteq_{Maddy}$ with a transitive interpretation relation, is not acceptable.
- see Hamkins (2013).

UNIVERSITÄT SALZBURG

Hamkins' counterexamples

- The relation $\trianglelefteq_{Maddy}$ defined by Maddy has the following structural problems:
  - the disjunction is not an upper bound;
  - it is not transitive.
- Hamkins' solution, viz. to replace $\trianglelefteq_{Maddy}$ with a transitive interpretation relation, is not acceptable.
- see Hamkins (2013).

Structure of the Presentation

Section's outline

- Definitions to avoid Hamkins' counterexamples;
- The new, and final, argument in favour of the GM_H.

Section's outline

- Definitions to avoid Hamkins' counterexamples;
- The new, and final, argument in favour of the GM_H.

Structure of the Presentation

Possibly truncated interpretation τ (1)

Let $\tau = \langle \delta, \epsilon \rangle$ be a translation and *rk* be the Mirimanoff rank function. Then we write $\delta_\alpha(y)$ for $\delta(y) \wedge rk(y) < \alpha$ and τ_α for $\langle \delta_\alpha, \epsilon \rangle$.

Possibly truncated interpretation

Let S and T – where T is $BST + A_0, \ldots, A_n$ – be set theories. A translation $\tau = \langle \delta, \epsilon \rangle$ is a *possibly truncated interpretation* of T in S if and only if

- $S \vdash \phi^\tau$ for every axiom of T;
- $S \vdash A^\tau \vee (\exists\kappa[Inacc(\kappa)]^\tau \wedge A^{\tau_\kappa})$

Possibly truncated interpretation τ (1)

Let $\tau = \langle \delta, \epsilon \rangle$ be a translation and *rk* be the Mirimanoff rank function. Then we write $\delta_\alpha(y)$ for $\delta(y) \wedge rk(y) < \alpha$ and τ_α for $\langle \delta_\alpha, \epsilon \rangle$.

Possibly truncated interpretation

Let S and T – where T is $BST + A_0, \ldots, A_n$ – be set theories. A translation $\tau = \langle \delta, \epsilon \rangle$ is a *possibly truncated interpretation* of T in S if and only if

- $S \vdash \phi^\tau$ for every axiom of T;
- $S \vdash A^\tau \vee (\exists \kappa [Inacc(\kappa)^\tau \wedge A^{\tau_\kappa}]$.

Possibly truncated interpretation τ (1)

Let $\tau = \langle \delta, \epsilon \rangle$ be a translation and *rk* be the Mirimanoff rank function. Then we write $\delta_\alpha(y)$ for $\delta(y) \wedge rk(y) < \alpha$ and τ_α for $\langle \delta_\alpha, \epsilon \rangle$.

Possibly truncated interpretation

Let S and T – where T is $BST + A_0, \ldots, A_n$ – be set theories. A translation $\tau = \langle \delta, \epsilon \rangle$ is a *possibly truncated interpretation* of T in S if and only if

- $S \vdash \phi^\tau$ for every axiom of T;
- $S \vdash A^\tau \vee (\exists \kappa [Inacc(\kappa)^\tau \wedge A^{\tau_\kappa}]$.

Possibly truncated interpretation τ (1)

Let $\tau = \langle \delta, \epsilon \rangle$ be a translation and *rk* be the Mirimanoff rank function. Then we write $\delta_\alpha(y)$ for $\delta(y) \wedge rk(y) < \alpha$ and τ_α for $\langle \delta_\alpha, \epsilon \rangle$.

Possibly truncated interpretation

Let S and T – where T is $BST + A_0, \ldots, A_n$ – be set theories. A translation $\tau = \langle \delta, \epsilon \rangle$ is a *possibly truncated interpretation* of T in S if and only if

- $S \vdash \phi^\tau$ for every axiom of T;
- $S \vdash A^\tau \vee (\exists \kappa [Inacc(\kappa)^\tau \wedge A^{\tau_\kappa}]$.

Possibly truncated interpretations τ (2)

Let S and T – T being $BST + A_0, \ldots, A_n$ – be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be a possibly truncated interpretation of T in S.

- τ is a *possibly truncated* $\in$*-intepretation* of T in S if and only if $S \vdash \forall x \forall y [(x \in y)^\tau \to x \in y]$;
- τ is a *possibly truncated transitive interpretation* of T in S if and only if τ is a possibly truncated $\in$-intepretation and $S \vdash \forall x \forall y [(x \in y \wedge \delta(y)) \to \delta(x)]$;
- τ is a *possibly truncated inner model intepretation* of T in S is and only if τ is a possibly truncated transitive interpretation and $S \vdash \forall \alpha [\delta(\alpha)]$. If there is such τ we write $T \trianglelefteq_{ptim} S$.

Possibly truncated interpretations τ (2)

Let S and T – T being $BST + A_0, \ldots, A_n$ – be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be a possibly truncated interpretation of T in S.

- τ is a *possibly truncated $\in$-intepretation* of T in S if and only if $S \vdash \forall x \forall y[(x \in y)^\tau \to x \in y]$;
- τ is a *possibly truncated transitive interpretation* of T in S if and only if τ is a possibly truncated $\in$-intepretation and $S \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \to \delta(x)]$;
- τ is a *possibly truncated inner model interpretation* of T in S is and only if τ is a possibly truncated transitive interpretation and $S \vdash \forall \alpha[\delta(\alpha)]$. If there is such τ we write $T \trianglelefteq_{ptim} S$.

Possibly truncated interpretations τ (2)

Let S and T – T being $BST + A_0, \ldots, A_n$ – be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be a possibly truncated interpretation of T in S.

- τ is a *possibly truncated $\in$-intepretation* of T in S if and only if $S \vdash \forall x \forall y[(x \in y)^\tau \rightarrow x \in y]$;
- τ is a *possibly truncated transitive interpretation* of T in S if and only if τ is a possibly truncated $\in$-intepretation and $S \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$;
- τ is a *possibly truncated inner model interpretation* of T in S is and only if τ is a possibly truncated transitive interpretation and $S \vdash \forall \alpha[\delta(\alpha)]$. If there is such τ we write $T \trianglelefteq_{ptim} S$.

Possibly truncated interpretations τ (2)

Let S and T – T being $BST + A_0, \ldots, A_n$ – be set theories and let $\tau = \langle \delta, \epsilon \rangle$ be a possibly truncated interpretation of T in S.

- τ is a *possibly truncated* $\in$*-intepretation* of T in S if and only if $S \vdash \forall x \forall y[(x \in y)^\tau \rightarrow x \in y]$;
- τ is a *possibly truncated transitive interpretation* of T in S if and only if τ is a possibly truncated $\in$-intepretation and $S \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$;
- τ is a *possibly truncated inner model interpretation* of T in S is and only if τ is a possibly truncated transitive interpretation and $S \vdash \forall \alpha[\delta(\alpha)]$. If there is such τ we write $T \trianglelefteq_{ptim} S$.

Maximizing possibly truncated interpretation

Let S and T – T being $BST + A_0, \ldots, A_n$ – be set theories. A translation $\tau = \langle \delta, \epsilon \rangle$ is a maximizing possibly truncated interpretation of T in S if and only if

- $S \vdash \phi^{\tau}$ for every axiom ϕ of T;
- $S \vdash (A^{\tau} \wedge NewIso(\tau)) \vee (\exists\kappa[Inacc(\kappa)^{\tau} \wedge (A^{\tau_\kappa} \wedge NewIso(\tau_\kappa))]))$.

Maximizing possibly truncated interpretation

Let S and T – T being $BST + A_0, \ldots, A_n$ – be set theories. A translation $\tau = \langle \delta, \epsilon \rangle$ is a maximizing possibly truncated interpretation of T in S if and only if

- $S \vdash \phi^\tau$ for every axiom ϕ of T;
- $S \vdash (A^\tau \wedge NewIso(\tau) \vee (\exists \kappa [Inacc(\kappa)^\tau \wedge (A^{\tau_\kappa} \wedge NewIso(\tau_\kappa))]))$.

Maximizing possibly truncated interpretation

Let S and T – T being $BST + A_0, \ldots, A_n$ – be set theories. A translation $\tau = \langle \delta, \epsilon \rangle$ is a maximizing possibly truncated interpretation of T in S if and only if

- $S \vdash \phi^\tau$ for every axiom ϕ of T;
- $S \vdash (A^\tau \wedge NewIso(\tau) \vee (\exists \kappa [Inacc(\kappa)^\tau \wedge (A^{\tau_\kappa} \wedge NewIso(\tau_\kappa))]))$.

Maddy* interpretation

Let S and T – T being $BST + A_0, \ldots, A_n$ – be set theories and let τ be a possibly truncated interpretation of T in S. τ is a *Maddy** interpretation if and only if

- is a possibly truncated inner model interpretation;
- is a maximizing possibly truncated interpretation.

If this is the case, we say that S *Maddy** interpretates T and we write $T \trianglelefteq_{Maddy^*} S$.

Maddy* interpretation

Let S and T – T being $BST + A_0, \ldots, A_n$ – be set theories and let τ be a possibly truncated interpretation of T in S. τ is a *Maddy** interpretation if and only if

- is a possibly truncated inner model interpretation;
- is a maximizing possibly truncated interpretation.

If this is the case, we say that S *Maddy** interpretates T and we write $T \trianglelefteq_{Maddy^*} S$.

Maddy* interpretation

Let S and T – T being $BST + A_0, \dots, A_n$ – be set theories and let τ be a possibly truncated interpretation of T in S. τ is a *Maddy** interpretation if and only if

- is a possibly truncated inner model interpretation;
- is a maximizing possibly truncated interpretation.

If this is the case, we say that S *Maddy** interpretates T and we write $T \trianglelefteq_{Maddy^*} S$.

Maddy* interpretation

Let S and T – T being $BST + A_0, \dots, A_n$ – be set theories and let τ be a possibly truncated interpretation of T in S. τ is a *Maddy** interpretation if and only if

- is a possibly truncated inner model interpretation;
- is a maximizing possibly truncated interpretation.

If this is the case, we say that S *Maddy** interpretates T and we write $T \trianglelefteq_{Maddy^*} S$.

Solving Hamkins' problems

Proposition [Incurvati, Löwe]

Both $\trianglelefteq_{ptim}$ and $\trianglelefteq_{Maddy^*}$ are monotone.

Proposition [Incurvati, Löwe]

If $T \trianglelefteq_{ptim} BST + A$ and $T \trianglelefteq_{ptim} BST + B$, then $T \trianglelefteq_{ptim} BST + A \vee B$. The same holds for $\trianglelefteq_{Maddy^*}$.

Proposition [Incurvati, Löwe]

Both $\trianglelefteq_{ptim}$ and $\trianglelefteq_{Maddy^*}$ are transitive.

Solving Hamkins' problems

Proposition [Incurvati, Löwe]

Both $\trianglelefteq_{ptim}$ and $\trianglelefteq_{Maddy^*}$ are monotone.

Proposition [Incurvati, Löwe]

If $T \trianglelefteq_{ptim} BST + A$ and $T \trianglelefteq_{ptim} BST + B$, then $T \trianglelefteq_{ptim} BST + A \vee B$. The same holds for $\trianglelefteq_{Maddy^*}$.

Proposition [Incurvati, Löwe]

Both $\trianglelefteq_{ptim}$ and $\trianglelefteq_{Maddy^*}$ are transitive.

Solving Hamkins' problems

Proposition [Incurvati, Löwe]

Both $\trianglelefteq_{ptim}$ and $\trianglelefteq_{Maddy^*}$ are monotone.

Proposition [Incurvati, Löwe]

If $T \trianglelefteq_{ptim} BST + A$ and $T \trianglelefteq_{ptim} BST + B$, then $T \trianglelefteq_{ptim} BST + A \vee B$. The same holds for $\trianglelefteq_{Maddy^*}$.

Proposition [Incurvati, Löwe]

Both $\trianglelefteq_{ptim}$ and $\trianglelefteq_{Maddy^*}$ are transitive.

Structure of the Presentation

1. Introduction
2. The Generic Multiverse with a core
3. A first argument for the GM_H
   - Restrictiveness
   - Refining of the definitions
4. A further refinement
   - Solving Hamkins' problems
   - A second argument for the GM_H
5. Conclusions

A new proposition

Proposition

$ZFC \trianglelefteq_{Maddy^*} GM_H$.

- GM_H $Maddy^*$ interpretates ZFC if and only if there exists a translation τ of ZFC in GM_H that is a

A new proposition

Proposition

$ZFC \trianglelefteq_{Maddy^*} GM_H$.

- GM_H $Maddy^*$ interpretates ZFC if and only if there exists a translation τ of ZFC in GM_H that is a
  1. a possible truncated inner model interpretation of ZFC in GM_H;
  2. a maximizing possibly truncated interpretation of ZFC in GM_H.

A new proposition

Proposition

$ZFC \trianglelefteq_{Maddy^*} GM_H$.

- GM_H $Maddy^*$ interpretates ZFC if and only if there exists a translation τ of ZFC in GM_H that is a
  1. a possible truncated inner model interpretation of ZFC in GM_H;
  2. a maximizing possibly truncated interpretation of ZFC in GM_H.

UNIVERSITÄT SALZBURG

A new proposition

Proposition

$ZFC \trianglelefteq_{Maddy^*} GM_H$.

- GM_H $Maddy^*$ interpretates ZFC if and only if there exists a translation τ of ZFC in GM_H that is a
  1. a possible truncated inner model interpretation of ZFC in GM_H;
  2. a maximizing possibly truncated interpretation of ZFC in GM_H.

Sketch of the proof (1)

1. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff ZFC \trianglelefteq_{ptim} GM_H$
   - τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;
     - τ is a possible truncated $\in$ - interpretation of ZFC in GM_H
2. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff \exists \tau$ such that

Sketch of the proof (1)

1. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff ZFC \trianglelefteq_{ptim} GM_H$
   - τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;
     - τ is a possible truncated $\in$ - interpretation of ZFC in GM_H iff $GM_H \vdash \forall x \forall y[(x \in y)^\tau \rightarrow x \in y]$;
     - $GM_H \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$;
   - $GM_H \vdash \forall \alpha[\delta(\alpha)]$
2. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff \exists \tau$ such that

Sketch of the proof (1)

1. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff ZFC \trianglelefteq_{ptim} GM_H$
   - τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;
     - τ is a possible truncated $\in$ - interpretation of ZFC in GM_H iff $GM_H \vdash \forall x \forall y[(x \in y)^\tau \rightarrow x \in y]$;
     - $GM_H \vdash \forall x \forall y[(x \in y \wedge \theta(y)) \rightarrow \theta(x)]$;
   - $GM_H \vdash \forall \alpha[\theta(\alpha)]$
2. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff \exists \tau$ such that

Sketch of the proof (1)

1. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff ZFC \trianglelefteq_{ptim} GM_H$
   - τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;
     - τ is a possible truncated $\in$ - interpretation of ZFC in GM_H iff $GM_H \vdash \forall x \forall y[(x \in y)^\tau \rightarrow x \in y]$;
     - $GM_H \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$.
   - $GM_H \vdash \forall \alpha[\delta(\alpha)]$
2. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff \exists \tau$ such that

Sketch of the proof (1)

1. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff ZFC \trianglelefteq_{ptim} GM_H$
   - τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;
     - τ is a possible truncated $\in$ - interpretation of ZFC in GM_H iff $GM_H \vdash \forall x \forall y[(x \in y)^\tau \to x \in y]$;
     - $GM_H \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \to \delta(x)]$
   - $GM_H \vdash \forall \alpha[\delta(\alpha)]$.

Sketch of the proof (1)

1. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff ZFC \trianglelefteq_{ptim} GM_H$
   - τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;
     - τ is a possible truncated $\in$ - interpretation of ZFC in GM_H iff $GM_H \vdash \forall x \forall y[(x \in y)^\tau \rightarrow x \in y]$;
     - $GM_H \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$.
   - $GM_H \vdash \forall \alpha[\delta(\alpha)]$.
2. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff \exists \tau$ such that
   - $GM_H \vdash \varphi^\tau$ for every axiom φ of ZFC;
   - $GM_H \vdash (A^\tau \wedge NewIso(\tau) \vee (\exists \kappa[Inacc(\kappa)^\tau \wedge (A^{\tau\kappa} \wedge NewIso(\tau_\kappa))])$

Sketch of the proof (1)

1. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff ZFC \trianglelefteq_{ptim} GM_H$
   - τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;
     - τ is a possible truncated $\in$ - interpretation of ZFC in GM_H iff $GM_H \vdash \forall x \forall y[(x \in y)^\tau \rightarrow x \in y]$;
     - $GM_H \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$.
   - $GM_H \vdash \forall \alpha[\delta(\alpha)]$.
2. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff \exists \tau$ such that
   - $GM_H \vdash \phi^\tau$ for every axiom ϕ of ZFC;

Sketch of the proof (1)

1. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff ZFC \trianglelefteq_{ptim} GM_H$
   - τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;
     - τ is a possible truncated $\in$ - interpretation of ZFC in GM_H iff $GM_H \vdash \forall x \forall y[(x \in y)^\tau \rightarrow x \in y]$;
     - $GM_H \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$.
   - $GM_H \vdash \forall \alpha[\delta(\alpha)]$.
2. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff \exists \tau$ such that
   - $GM_H \vdash \phi^\tau$ for every axiom ϕ of ZFC;
   - $GM_H \vdash (A^\tau \wedge NewIso(\tau) \vee (\exists \kappa[Inacc(\kappa)^\tau \wedge (A^{\tau_\kappa} \wedge NewIso(\tau_\kappa))]))$.

Sketch of the proof (1)

1. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff ZFC \trianglelefteq_{ptim} GM_H$
   - τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;
     - τ is a possible truncated $\in$ - interpretation of ZFC in GM_H iff $GM_H \vdash \forall x \forall y[(x \in y)^\tau \rightarrow x \in y]$;
     - $GM_H \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$.
   - $GM_H \vdash \forall \alpha[\delta(\alpha)]$.
2. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff \exists \tau$ such that
   - $GM_H \vdash \phi^\tau$ for every axiom ϕ of ZFC;
   - $GM_H \vdash (A^\tau \wedge NewIso(\tau)) \vee (\exists \kappa[Inacc(\kappa)^\tau \wedge (A^{\tau_\kappa} \wedge NewIso(\tau_\kappa))]])$.

Sketch of the proof (1)

1. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff ZFC \trianglelefteq_{ptim} GM_H$
   - τ is a possibly truncated transitive interpretation of ZFC in GM_H iff;
     - τ is a possible truncated $\in$ - interpretation of ZFC in GM_H iff $GM_H \vdash \forall x \forall y[(x \in y)^\tau \rightarrow x \in y]$;
     - $GM_H \vdash \forall x \forall y[(x \in y \wedge \delta(y)) \rightarrow \delta(x)]$.
   - $GM_H \vdash \forall \alpha[\delta(\alpha)]$.
2. $ZFC \trianglelefteq_{Maddy^*} GM_H \iff \exists \tau$ such that
   - $GM_H \vdash \phi^\tau$ for every axiom ϕ of ZFC;
   - $GM_H \vdash (A^\tau \wedge NewIso(\tau) \vee (\exists \kappa[Inacc(\kappa)^\tau \wedge (A^{\tau_\kappa} \wedge NewIso(\tau_\kappa))]))$.

UNIVERSITÄT SALZBURG

The final proposition

Proposition

ZFC is strongly restrictive over GM_H.

- GM_H strongly maximizes over ZFC iff
  1. GM_H weakly maximizes over ZFC iff
     - $ZFC \trianglelefteq_{Maddy^*} GM_H$;
     - there is no set theory T that extends ZFC such that $GM_H \trianglelefteq_{Maddy^*} T$.
  2. $GM_H \cup ZFC$ is inconsistent;

The final proposition

Proposition

ZFC is strongly restrictive over GM_H.

- GM_H strongly maximizes over ZFC iff
  1. GM_H weakly maximizes over ZFC iff
     - $ZFC \trianglelefteq_{Maddy^*} GM_H$;
     - there is no set theory T that extends ZFC such that $GM_H \trianglelefteq_{Maddy^*} T$
  2. $GM_H \cup ZFC$ is inconsistent;

UNIVERSITÄT SALZBURG

The final proposition

Proposition

ZFC is strongly restrictive over GM_H.

- GM_H strongly maximizes over ZFC iff
  1. GM_H weakly maximizes over ZFC iff
     - $ZFC \trianglelefteq_{Maddy^*} GM_H$;
     - there is no set theory T that extends ZFC such that $GM_H \trianglelefteq_{Maddy^*} T$
  2. $GM_H \cup ZFC$ is inconsistent;

The final proposition

Proposition

ZFC is strongly restrictive over GM_H.

- GM_H strongly maximizes over ZFC iff
  1. GM_H weakly maximizes over ZFC iff
     - $ZFC \trianglelefteq_{Maddy^*} GM_H$;
     - there is no set theory T that extends ZFC such that $GM_H \trianglelefteq_{Maddy^*} T$
  2. $GM_H \cup ZFC$ is inconsistent;

The final proposition

Proposition

ZFC is strongly restrictive over GM_H.

- GM_H strongly maximizes over ZFC iff
  1. GM_H weakly maximizes over ZFC iff
     - $ZFC \trianglelefteq_{Maddy^*} GM_H$;
     - there is no set theory T that extends ZFC such that $GM_H \trianglelefteq_{Maddy^*} T$
  2. $GM_H \cup ZFC$ is inconsistent;

UNIVERSITÄT SALZBURG

Structure of the Presentation

Conclusions

- Assuming MAXIMIZE, the GM_H fares better than the classic ZFC, since it provides more isomorphisms types.
- This is true of every type of multiverse.
- But the GM_H can interpret ZFC;
- In conclusion, the GM_H seems naturalistic better than ZFC.

UNIVERSITÄT SALZBURG

Conclusions

- Assuming MAXIMIZE, the GM_H fares better than the classic ZFC, since it provides more isomorphisms types.
- This is true of every type of multiverse.
- But the GM_H can interpret ZFC;
- In conclusion, the GM_H seems naturalistic better than ZFC.

Conclusions

- Assuming MAXIMIZE, the GM_H fares better than the classic ZFC, since it provides more isomorphisms types.
- This is true of every type of multiverse.
- But the GM_H can interpret ZFC;
- In conclusion, the GM_H seems naturalistic better than ZFC.

Conclusions

- Assuming MAXIMIZE, the GM_H fares better than the classic ZFC, since it provides more isomorphisms types.
- This is true of every type of multiverse.
- But the GM_H can interpret ZFC;
- In conclusion, the GM_H seems naturalistic better than ZFC.