A naturalistic case in favour of the Generic Multiverse with a core

May 26, 2018

In this paper, I compare the Generic Multiverse with a core (henceforth, GM_H) with the classical set theory ZFC, making use of the principle MAXIMIZE introduced by Maddy (1997). This principle states that, since the aim of set theory is to represent all the known mathematics within a single theory, it should *maximize* the range of available isomorphism types. This is particular important for mathematics, since isomorphisms make it is possible to import methods and results from a mathematical field to another. I argue that the classic set theory ZFC is *restrictive* over the GM_H, that is, the GM_H *strongly maximizes* over ZFC in the sense that it provides a wide range of isomorphism types that are not available in ZFC.

I briefly define the GM_H as the multiverse with a common core of truths, shared between all the universes of the multiverse. A universe in this multiverse is a model of a certain set of axioms of set theory (for example $ZFC + V = L$ or $ZF + AD$), while the core is a set of propositions satisfied in every universe of the multiverse. Obviously in the multiverse there is also the universe that satisfies only the propositions in the core (that is, the core has a model that is part of the multiverse). All the other universes are extensions of this core: they satisfy all that is true in it, and more. For example, if the core is the the intended model of ZFC, the multiverse includes a model of $ZFC + V = L$ and a model of $ZFC+$ "$0^{\#}$ exists" (see Steel (2014)).

The GM_H thus defined strongly maximizes over ZFC: there is no theory T extending ZFC that properly maximizes over the GM_H and the GM_H inconsistently maximizes over ZFC. This means that the GM_H provides structures that cannot be satisfied by ZFC, even if properly extended. To see this, assume that the core of the GM_H is ZF^- (set theory minus the Axiom of Foundation). From this core we can build a multiverse in which, among others, there is a universe for ZFC and a universe for $ZF + AD$. In this multiverse one can have *both* the Axiom of Choice (provided by ZFC) *and* a full Axiom of Determinacy (provided by $ZF + AD$). Determinacy and Choice are actually incompatible, but they can coexist in the GM_H. Hence, the GM_H, unlike the intended model of ZFC, can include all the structures based on Determinacy. That is, the GM_H provides a new isomorphism type, i.e. it proves the existence of a structure that is not isomorphic to anything in ZFC.

Furthermore, the GM_H also provides what Maddy calls a *fair interpretation* of ZFC, i.e. the GM_H validates all the axioms of ZFC (this is because ZFC is part of the multiverse) and one can build natural models, inner models, and truncations of proper class of inner models at inaccessible levels of ZFC in the GM_H.

I conclude that, assuming MAXIMIZE, the GM_H is better justified than ZFC, since it provides more isomorphism types and it can fairly interpret ZFC itself.

References

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